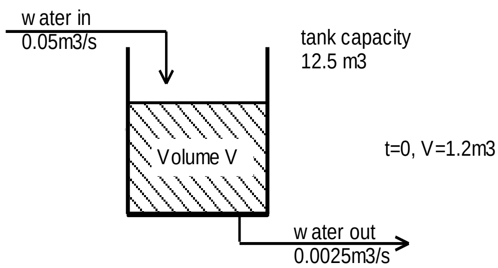
A Leaky Tank

Prior knowledge - Differentiation of polynomials, finding stationary points and intercepts.

Task:
A 12.5 m^3 tank is being filled at a rate of 0.05 m^3/s. The moment the tank reaches 120m^3 of water a bottom leak forms and gets progressively worse with time. The rate of leakage can be approximated as 0.0025t (m^3/s), where t is the time in seconds from the moment the leak begins.

1. Write a differential equation for the rate the volume changes (dV / dt). Give initial conditions for your equation.
2. Give an expression for V (solve for V)
3. Graph V versus t, and explain what it means, and find any other critical information.

The solutions:
Conditions: Time is taken from the moment the leak <u>begins</u> : i.e.: when $t = 0$, $V = 0$



1. $\dfrac{dV}{dt}$ = *rate water in - rate water out*

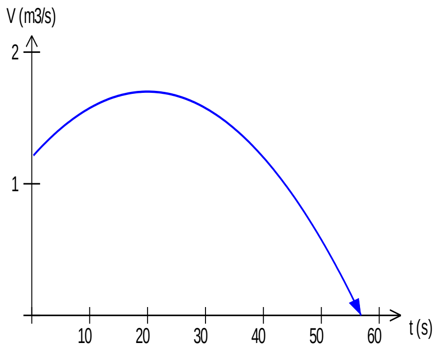
   $= 0.05 - 0.0025t$

2. Integrating $V = 0.05t - 0.00125t^2 + C$

   Substituting in to find C: $t = 0$, $V = 0$, we get $C = 1.2$

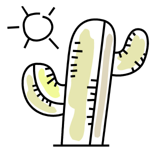
   Thus $V = 0.05t - 0.00125t^2 + 1.2$

3.



The graph indicates that the water initially rises in the tank, as the leak gets larger the tank begins to drain. The *maximum* volume can be found to be 1.7m^3, well below the capacity of 12.5m^3. At 57 seconds the tanks is completely dry *(x-intercept)*, and the function gives negative values, but the volume actually remains zero so the actual solution for V is a piece-wise function

$$V = \begin{cases} 1.2 + 0.05t - 0.00125t^2 & 0 \le t \le 57s \\ 0 & t > 57s \end{cases}$$



Will the City Reservoir dry up?

A more advanced problem similar to the above. Involves integrating e^x

TASK: The water level in a city reservoir has been decreasing steadily during a dry spell, and there is concern it could dry up in the next 60 days if the drought continues. The local water company estimates the consumption rate is approximately 10^7 L/day and the State Conservation Service estimates the rainfall and stream drainage into the reservoir coupled with evaporation from the reservoir should yield a net water input rate of $10^6 exp(-t/100)$ L/day, where t is time in days from the beginning of the drought, at which time the reservoir contained 10^9 L of water.

Solutions: let V = volume in reservoir (Litres), t = time (days)

$\dfrac{dV}{dt} = 10^6 e^{-\frac{t}{100}} - 10^7$, integrate to get $V = -10^8 e^{-\frac{t}{100}} - 10^7 t + 10^9 + 10^8$ where $t = 0$, $V = 10^9$,
Evaluating V when $t = 60$ days, we get $V = 4.45 \times 10^8$ L

(Adapted from: Elementary Principles of Chemical Processes 3rd edition, Felder and Rousseau, pp547-549)